The Ratio Theorem

Finding the midpoint of a line is a special case of dividing a line in a given ratio.

Example.

Find the point C which divides the line AB, with endpoints A(2, 0) and B (8, 9), in the ratio AC:CB=1:2.

In other words we need to find the point C which is $^1/_3$ of the way along the line from A to B.

In general, for A:(p, q) and B:(r, s) and ratio AC:CB=m:n then C is the point with coordinates $\left(\frac{np+mr}{m+n}, \frac{nq+ms}{m+n}\right)$

So in our example we have C:$\left(\frac{4+8}{3}, \frac{0+9}{3}\right)$ i.e. the point (4,3)