Lesson plan for introducing the equation of a circle.

Aim: Making the link between equations of circles and Pythagoras and using this to find the equations of circles. In this lesson the equation of any circle will be in the form $(x-a)^2+(y-b)^2=r^2$; completing the square will be left until next lesson (when http://www.meidistance.co.uk/js/GeomCircleEqun.html might be useful).

By the end of the lesson students should be confident with

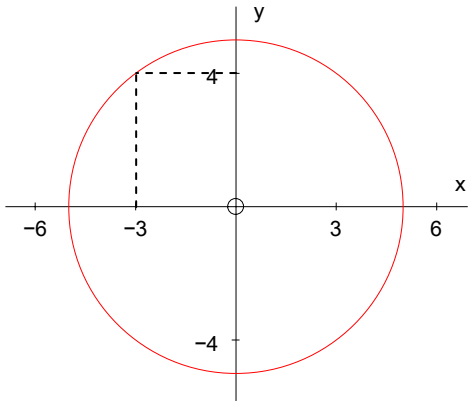
- finding the equation of a circle from its graph;
- drawing a circle given its equation;
- appreciating the connection between r, a and b when a circle touches, crosses or doesn't cross each axis.

Textbook reference
AS Core Mathematics: Chapter 2 Pages 61-67.
Additional Mathematics: Chapter 5, pages 96-99.

In the next lesson students will look at the expanded forms of the equation of a circle with the need to complete the square.

Introduction. (20 minutes)

Using the flash resource http://www.meidistance.co.uk/flash/ascgc2mg.html display the circle centred on the origin with radius 5.



Point out that $(-3,4)$ appears to lie on the circle.

- Is this true?
- How do you know it is exactly on the circle?
- What other points lie on the circle,
  other than $(0,\pm5)$ and $(\pm5,0)$?

This should allow you to discuss Pythagoras and then:

"So the equation of the circle is $x^2+y^2=5^2$." Now click on 'show equation'.

- Substitute $x=3$ into this equation. What is y?

There are two values of y. This is probably the first time students have come across an equation containing y^2 so you may want to discuss this and how it means that there can be two points on the curve corresponding to one value of x.

Now 'hide equation' and display the circle centred on the origin with radius 6.

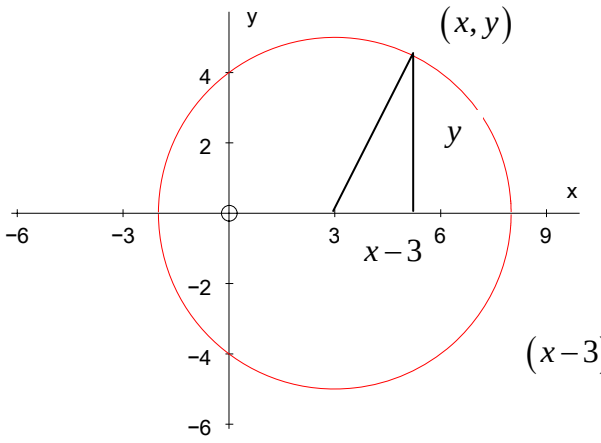
- What's the equation of this circle?

When you're happy that they've got the idea, 'hide equation' and drag the centre of the circle to the point $(3,0)$ and set the radius to 5.

- What is the equation of this circle?

If they come up with the wrong answers ask if obvious points such as $(-2,0)$ and $(3,5)$ lie on their circle.

To reinforce where the equation comes from draw a sketch:



$(x-3)^2+y^2=5^2$

Main activity. (30 minutes)

Hand out the worksheet below, cut up. In pairs, students should find a possible equation for each of the nine circles and write each on a separate piece of card. They then swap their nine circles and nine equations with those of a neighbouring pair. Each pair of students then matches their neighbours' cards up into pairs. Any points for debate can be discussed by the class and Autograph used to check answers.

Further cards can be found at http://www.meidistance.co.uk/pdf/tpc1_ct09.pdf

Early finishers could work on http://www.meidistance.co.uk/pdf/tpc1_cq05.pdf

Plenary (10 minutes)

Using mini-whiteboards:

- Give me the equation of a circle of radius 6.
- Give me an example of an equation of a circle centre (-1, 3).
- Give me the equation of the circle with radius 4 and centre (4, -5).
- Give me an equation of a circle which doesn't cross either axis.
- Give me an equation of a circle which passes through the origin.

Point to the central region in the Venn diagram in http://www.meidistance.co.uk/pdf/tpc1_cq05.pdf and ask for the equation of a circle satisfying these three conditions.
Repeat for other regions.

Homework:
Either complete the Venn diagram exercise or from the textbook:
AS Core Mathematics: Ex 2E All questions (even question 5) are possible.
Additional Mathematics: Ex 5E