Coordinate geometry: points of intersection: a circle and a line

A flash resource for use with this topic can be found here

Below are some suggestions how you might use this in a revision lesson. You are encouraged to think of other questions.

First of all, to familiarise yourself with the features, use the arrows to change the radius of the circle and the equation of the line, drag the centre of the circle to change it's position and 'show points of intersection' to find the co-ordinates of the point where the line crosses the circle.

This resource could be used to revise:

- Equations of straight lines;
- Equations of circles;
- Equations of the diameters of a circle;
- Tangents to circles.

Although other resources in this series are more useful in teaching and revising the first two topics individually, this allows you to look at several topics together.

Questions to ask your students.

After each question you can check students' answers using this flash resource.

Lines.

Set $r = 0$ so that the circle disappears.
"On your whiteboards write down the equation of the line:

- with gradient 2 which passes through the origin;
- with gradient -2 which passes through the point $(0, 2)$;
- with gradient -3 which passes through the point $(6, 0)$;
- with gradient 3 which passes through the point $(2, 2)$;
- which is parallel to $y + x = 4$ and passes through $(-3, 1)$;
- which is perpendicular to $y + x = 4$ and passes through $(0, 1)$;
- which passes through $(0, 2)$ and $(2, 8)$;
- which passes through $(-3, -3)$ and $(-3, 6)$;
- which passes through $(0, 2)$ and $(2, 8)$;
- which passes through $(-2, 0)$ and $(0, -4)$.

There should be the opportunity to discuss different forms of the equation of a line.

Circles.

Set the equation of the line to $y = 0$ so that the line effectively disappears.
“On your whiteboards write down:

- the coordinates of a point in the third quadrant (not on either axis) which lies on the circle with equation $x^2 + y^2 = 25$;
- the (integer) coordinates of a point which lies on the circle with equation $x^2 + y^2 = 100$;
- the equation of the circle centre the origin, radius 4;
- the equation of the circle centre $(2,1)$, radius 3;
- the equation of the circle centre $(-3,2)$, radius 4;
- the equation of a circle which touches the $x-$axis;
- the equation of a circle which is completely within the fourth quadrant;
- the equation of a circle which passes through the origin and whose centre is on one of the axes;
- the equation of a circle which passes through the origin and whose centre is **not** on either of the axes;
- the equation of the circle you get after translating the circle $(x-1)^2 + (y+2)^2 = 4^2$ through the vector $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

Lines and circles.

Set the equation of the line to $y = x$, the circle to $(x-1)^2 + y^2 = 25$ and ‘show points of intersection’. Mention that the points of intersection are given to two decimal places.
“We’ll call the points of intersection P_1 and P_2. On your whiteboards work out:

- The distance $|P_1P_2|$;
- The equation of the line parallel to $y = x$ which is a diameter if the circle;
- The distance $|P_1P_2|$ for this line;
- The equation of the diameter perpendicular to this line;
- The equation of a diameter of the circle $(x-3)^2 + (y+2)^2 = 16$ (if they write $x = 3$ or $y = -2$ then ask for another);
- The equation of a line that does not pass through the origin but cuts the circle $(x-2)^2 + (y-2)^2 = 16$ in the second and fourth quadrants;
- The equation of a circle which has $y + 2x + 5 = 0$ as a diameter;
- The points of intersection of the line $y = x - 6$ and the circle $(x-1)^2 + (y-2)^2 = 25$.

You might want to discuss the difference between a finite diameter and an infinite line.

Follow up: Make a poster showing how you'd find the two tangents to the circle $(x-4)^2+(y-3)^2=9$ which pass through the origin. Note that, due to integer constraints, you cannot show these using the flash resource although you can compare the answer with the lines $y=3x$ and $y=4x$.

Solution:

Substitute $y=mx$ in $(x-4)^2+(y-3)^2=9$ and find the two values of m so that the resulting quadratic has repeated roots:

$$(x-4)^2+(mx-3)^2=9 \quad\Rightarrow\quad (1+m^2)x^2-(8+6m)x+16=0$$

$$"b^2=4ac" \quad\Rightarrow\quad (8+6m)^2=4\times(1+m^2)\times16$$

$$\Rightarrow \quad 0=4m(7m-24)$$

$$\Rightarrow \quad m=0,\ \frac{24}{7}$$