OCR Core 1

Coordinate Geometry

Section 2: Circles

Solutions to Exercise

1. (i) $(x-0)^2+(y-0)^2=6^2$

$x^2+y^2=36$

(ii) $(x-3)^2+(y-1)^2=5^2$

$x^2-6x+9+y^2-2y+1=25$

$x^2+y^2-6x-2y=15$

(iii) $(x+2)^2+(y-5)^2=1^2$

$x^2+4x+4+y^2-10y+25=1$

$x^2+y^2+4x-10y=-28$

(iv) $(x-0)^2+(y+4)^2=3^2$

$x^2+y^2+8y+16=9$

$x^2+y^2+8y=-7$

2. (i) $x^2+y^2=100=10^2$

Centre = (0, 0), radius = 10.

(ii) $(x-2)^2+\left(y-7\right)^2=16=4^2$

Centre = (2, 7), radius = 4

(iii) $(x+3)^2+\left(y-4\right)^2=4=2^2$

Centre = (-3, 4), radius = 2

(iv) $(x+4)^2+\left(y+5\right)^2=20$

Centre = (-4, -5), radius = $\sqrt{20}$

3. (i) $x^2+y^2+4x-5=0$

$x^2+4x+y^2-5=0$

$(x+2)^2-4+y^2-5=0$

$(x+2)^2+y^2=9$

Centre = (-2, 0), radius = 3.

(ii) $x^2+y^2-6x+10y+20=0$

$x^2-6x+y^2+10y+20=0$

$(x-3)^2-9+(y+5)^2-25+20=0$

$(x-3)^2+(y+5)^2=14$

Centre is (3, -5) and radius $=\sqrt{14}$

(iii) $x^2+y^2-2x-3y+3=0$

$x^2-2x+y^2-3y+3=0$

$(x-1)^2-1+(y-\frac{3}{2})^2-\frac{9}{4}+3=0$

$(x-1)^2+(y-\frac{3}{2})^2=1+\frac{9}{4}-3$

$(x-1)^2+(y-\frac{3}{2})^2=\frac{1}{4}$

Centre is $(1,\frac{3}{2})$ and radius $=\frac{1}{2}$.

4. Radius of circle $=\sqrt{(6-4)^2+(3-(-2))^2}=\sqrt{4+25}=\sqrt{29}$

Equation of circle is $(x-4)^2+(y+2)^2=29$

$x^2-8x+16+y^2+4y+4=29$

$x^2+y^2-8x+4y=20$

5. Centre of circle C is the midpoint of AB.

$$C=\left(\frac{2+6}{2},\frac{0+4}{2}\right)=(4,2)$$

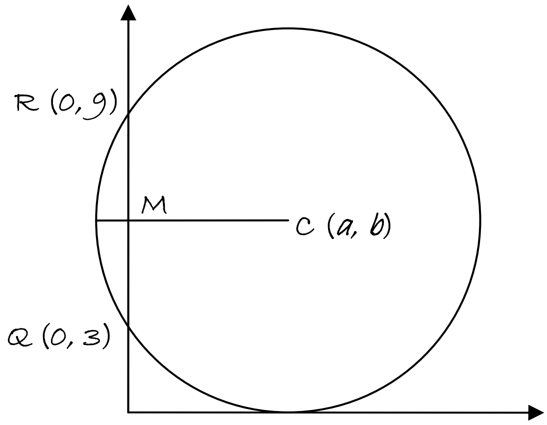
Radius of circle is distance AC $=\sqrt{(2-4)^2+(0-2)^2}=\sqrt{4+4}=\sqrt{8}$

Equation of circle is $(x-4)^2+(y-2)^2=8$

$x^2-8x+16+y^2-4y+4=8$

$x^2+y^2-8x-4y+12=0$

6.



The midpoint M of QR is (0, 6).
Since a diameter which passes through M is perpendicular to QR, then the line CM must be horizontal, and therefore b = 6.

Since the circle touches the x-axis, the radius of the circle must be b, i.e. 6.

The equation of the circle is therefore $(x-a)^2+(y-6)^2=6^2$

The circle passes through (0, 3), so $(0-a)^2+(3-6)^2=6^2$

$$a^2+9=36$$
$$a^2=27$$
$$a=\pm\sqrt{27}=\pm3\sqrt{3}$$

The equation of the circle is either $(x-3\sqrt{3})^2+(b-6)^2=36$

or $(x+3\sqrt{3})^2+(b-6)^2=36$.

7. (i) $x^2+y^2=8$

Substituting in $y=4-x$ gives $x^2+(4-x)^2=8$

$$x^2+16-8x+x^2=8$$
$$2x^2-8x+8=0$$
$$x^2-4x+4=0$$
$$(x-2)^2=0$$

The line meets the circle at just one point, so the line touches the circle and is therefore a tangent.

(ii) $x^2+y^2=25$

Substituting in $4y=3x-25 \quad \Rightarrow y=\dfrac{3x-25}{4}$

gives $x^2 + y^2 = 25$

$$x^2 + \left(\frac{3x-25}{4}\right)^2 = 25$$

$$x^2 + \frac{(3x-25)^2}{16} = 25$$

$$16x^2 + 9x^2 - 150x + 625 = 400$$

$$25x^2 - 150x + 225 = 0$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

The line meets the circle at just one point, so the line touches the circle and is therefore a tangent.

8. $x^2 + y^2 = 65$

$2y + x = 10 \quad \Rightarrow x = 10 - 2y$

Substituting in: $(10-2y)^2 + y^2 = 65$

$$100 - 40y + 4y^2 + y^2 = 65$$

$$5y^2 - 40y + 35 = 0$$

$$y^2 - 8y + 7 = 0$$

$$(y-1)(y-7) = 0$$

$$y = 1 \text{ or } y = 7$$

When $y = 1$, $x = 10 - 2\times 1 = 8$

When $y = 7$, $x = 10 - 2\times 7 = -4$

so P is (8, 1) and Q is (-4, 7)

Length PQ $= \sqrt{(8-(-4))^2 + (1-7)^2} = \sqrt{144+36} = \sqrt{180}$

9. (i) Gradient of PR $= \dfrac{7-6}{5-(-2)} = \dfrac{1}{7}$

Gradient of QR $= \dfrac{7-0}{5-6} = \dfrac{7}{-1} = -7$

Gradient of PR × gradient of QR $= \dfrac{1}{7} \times -7 = -1$

so PR and QR are perpendicular.

(ii) The angle in a semicircle is 90°, so PQ must be a diameter.

(iii) Since PQ is a diameter, the centre C of the circle is the midpoint of PQ.

$$C = \left(\frac{-2+6}{2}, \frac{6+0}{2}\right) = (2, 3)$$

Radius of circle = length CQ= $\sqrt{(6-2)^2+(0-3)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$

Equation of circle is $(x-2)^2+(y-3)^2=25$.

10. (i) $x^2+y^2=17$

(ii) Substituting x = -4 and y = -1: $x^2+y^2=(-4)^2+(-1)^2=16+1=17$

(iii) Gradient of radius OP $= \frac{-1-0}{-4-0} = \frac{1}{4}$

Tangent to circle at P is perpendicular to radius OP
so gradient of tangent = -4
Equation of tangent is $y-(-1)=-4(x-(-4))$

$$y+1=-4(x+4)$$
$$y+1=-4x-16$$
$$y+4x+17=0$$

(iv) $x+y=3 \quad \Rightarrow y=3-x$

Substituting into equation of circle: $x^2+(3-x)^2=17$

$$x^2+9-6x+x^2=17$$
$$2x^2-6x-8=0$$
$$x^2-3x-4=0$$
$$(x-4)(x+1)=0$$
$$x=4 \text{ or } x=-1$$

When x = 4, $y=3-4=-1$
When x = -1, $y=3-(-1)=4$
Coordinates of Q and R are (4, -1) and (-1, 4).

(v) Tangent is $y+4x+17=0$

Substituting in $y=3-x$ gives $(3-x)+4x+17=0$

$$20+3x=0$$
$$x=-\tfrac{20}{3}$$

When $x=-\frac{20}{3}$, $y=3+\frac{20}{3}=\frac{29}{3}$

Coordinates of S are $\left(\frac{20}{3},-\frac{11}{3}\right)$