OCR Core 1

Coordinate geometry

Section 1: Coordinates, points and lines

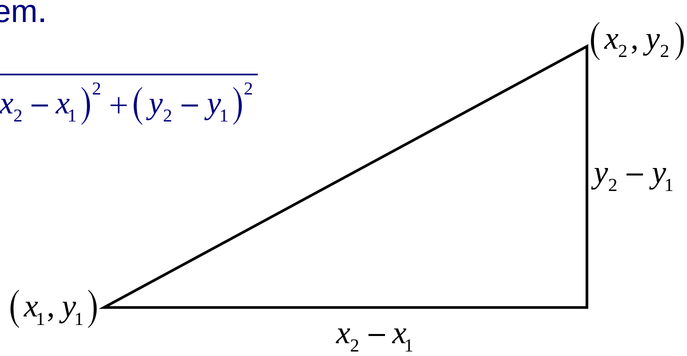
Notes and Examples

These notes contain sub-sections on:

Distances, mid-points and gradients

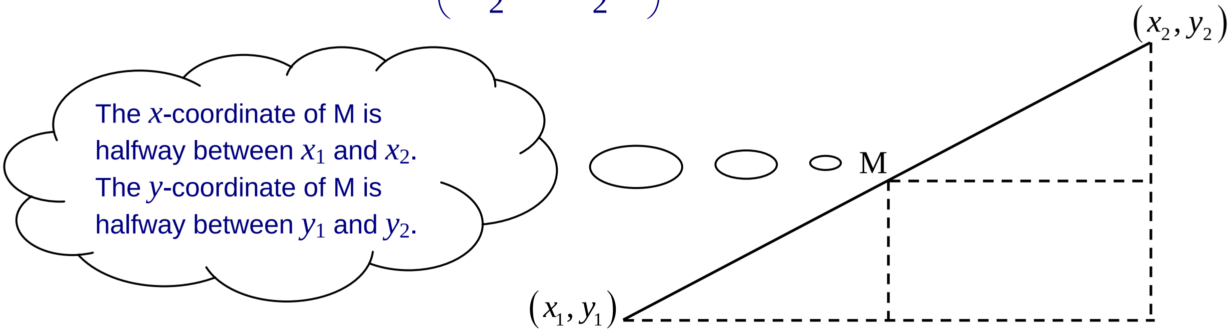
The length of a line joining two points (x_1, y_1) and (x_2, y_2) can be found using Pythagoras' Theorem.

Length $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



The midpoint of a line joining two points (x_1, y_1) and (x_2, y_2) is given by

Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



Example 1
A is the point (2, -6). B is the point (-3, 4).
Calculate
(i) the midpoint of AB
(ii) the length of AB.

Solution

Choose A as (x_1, y_1) and B as (x_2, y_2).

or vice versa, it will still give the same answer (***WHY***?)

(i) Midpoint is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

i.e. $\left(\frac{2+(-3)}{2}, \frac{-6+4}{2}\right)$

$= \left(\frac{-1}{2}, -1\right)$

(ii) The distance AB is given by

$$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$
$$= \sqrt{(2-(-3))^2+((-6)-4)^2}$$
$$= \sqrt{(5)^2+(-10)^2}$$
$$= \sqrt{25+100}$$
$$= \sqrt{125}$$

Note: The answer is often left like this if the square root is not exact. However since 125 = 25×5 then $\sqrt{125} = \sqrt{25}\sqrt{5} = 5\sqrt{5}$ is perhaps a simpler form.

To see more examples like these, try the Flash resources **Distance between two points** and **Midpoint of two points**.

For further practice in examples like the one above, try the interactive questions **The distance between two points** and **The midpoint between two points**.

You will have met gradients before at GCSE. To revise finding the gradient of a line from a diagram, use the interactive resource **The gradient of a line**. Remember that lines which go "downhill" have negative gradients.

To find the gradient of a straight line between two points (x_1, y_1) and (x_2, y_2), use the formula

$$\text{gradient} = \frac{y_2-y_1}{x_2-x_1}.$$

If two lines are parallel, they have the same gradient.

Example 2

P is the point (-3, 7). Q is the point (5, 1).

Calculate the gradient of PQ

Solution

Choose P as (x_1, y_1) and Q as (x_2, y_2).

$$\text{Gradient of PQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-7}{5-(-3)} = \frac{-6}{8} = -\frac{3}{4}$$

Notes:
(1) Draw a sketch and check that your answer is sensible (e.g. has negative gradient).
(2) Check that you get the same result when you choose Q as (x_1, y_1) and P as (x_2, y_2).

For more examples on gradient, look at the Flash resource **Gradient of a line**.

For further practice in examples like the one above, try the interactive questions **The gradient of a line between two points**.

The equation of a straight line

The equation of a straight line is often written in the form $y = mx + c$, where m is the gradient and c is the intercept with the y-axis.

Example 3

Find (i) the gradient and (ii) the y-intercept of the following straight-line equations.

(a) $5y = 7x - 3$ (b) $3x + 8y - 7 = 0$

Solution

(a) Rearrange the equation into the form $y = mx + c$.

$5y = 7x - 3$ becomes $y = \frac{7}{5}x - \frac{3}{5}$

so $m = \frac{7}{5}$ and $c = -\frac{3}{5}$

Note the minus sign

(i) The gradient is $\frac{7}{5}$

(ii) The y-intercept is $-\frac{3}{5}$.

(b) Rearrange the equation into the form $y = mx + c$.

$3x + 8y - 7 = 0$ becomes $8y = -3x + 7$

giving $y = -\frac{3}{8}x + \frac{7}{8}$

so $m = -\frac{3}{8}$ and $c = \frac{7}{8}$

Note the minus sign

(i) The gradient is $-\frac{3}{8}$

(ii) The y-intercept is $\frac{7}{8}$.

Sometimes you may need to sketch the graph of a line. A sketch is a simple diagram showing the line in relation to the origin. It should also show the coordinates of the points where it cuts one or both axes.

You can explore straight line graphs using the Flash resources **Equation of a line y = mx + c** and **Equation of a line ax + by + c = 0**.

Example 4
Sketch the lines (a) $5y = 7x - 3$ (b) $3x + 8y - 7 = 0$

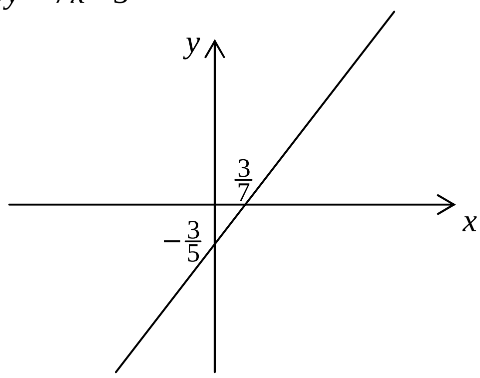
Solution
(a) From Example 3 you know that $5y = 7x - 3$ has gradient $\frac{7}{5}$ and y-intercept $-\frac{3}{5}$.

The graph therefore cuts the y-axis at $(0, -\frac{3}{5})$.

To find the point where the graph cuts the x-axis, substitute $y = 0$.

$0 = 7x - 3 \quad \Rightarrow x = \frac{3}{7}$, so the graph cuts the x-axis at $(\frac{3}{7}, 0)$.

Sketch of $5y = 7x - 3$



(b) From Example 3 you know that $3x + 8y - 7 = 0$ has gradient $-\frac{3}{8}$ and y-intercept $\frac{7}{8}$.

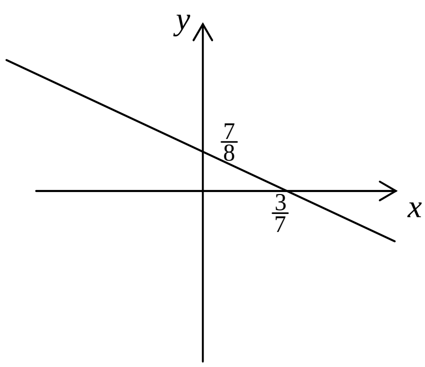
The graph therefore cuts the y-axis at $(0, \frac{7}{8})$.

To find the point where the graph cuts the x-axis, substitute $y = 0$.

$3x + 0 - 7 = 0 \quad \Rightarrow x = \frac{7}{3}$, so the graph cuts the x-axis at $(\frac{3}{7}, 0)$.

Notice that the gradient is negative, so the line will slope backwards.

Sketch of $3x + 8y - 7 = 0$



Sometimes you may need to find the equation of a line given certain information about it. If you are given the gradient and intercept, this is easy: you can simply use the form $y = mx + c$. However, more often you will be given the information in a different form, such as the gradient of the line and the coordinates of one point on the line (as in Example 5) or just the coordinates of two points on the line (as in Example 6).

In such cases you can use the alternative form of the equation of a straight line. For a line with gradient m passing through the point (x_1, y_1), the equation of the line is given by

$$y - y_1 = m(x - x_1).$$

Example 5

Find the equation of the line with gradient 2 and passing through (3, -1).

Solution

The equation of the line is $y - y_1 = m(x - x_1)$

$$\Rightarrow y - (-1) = 2(x - 3)$$
$$\Rightarrow y + 1 = 2x - 6$$
$$\Rightarrow y = 2x - 7$$

$m = 2$ and (x_1, y_1) is (3, -1)

You should check that the point (3, -1) satisfies your line. If it doesn't, you must have made a mistake!

You can see more examples like this using the Flash resource **Equation of a line y – y₁ =m(x – x₁)**.

In the next example, you are given the coordinates of two points on the line.

Example 6

P is the point (3, 8). Q is the point (-1, 5).

(i) Find the equation of PQ.

(ii) Find the equation of the line parallel to PQ which passes through the point (2, -3).

Solution

(i) Choose P as (x_1, y_1) and Q as (x_2, y_2).

Gradient of PQ = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-8}{-1-3} = \dfrac{-3}{-4} = \dfrac{3}{4}$

The equation of the line is $y - y_1 = m(x - x_1)$

$$y - 8 = \tfrac{3}{4}(x - 3)$$
$$4(y - 8) = 3(x - 3)$$
$$4y - 32 = 3x - 9$$
$$4y = 3x + 23$$

One method is to find the gradient and then use this value and one of the points in $y - y_1 = m(x - x_1)$

You should check that P and Q satisfy your line.

(ii) The line has gradient $\frac{3}{4}$ and passes through the point (2, -3).

Equation of line is $y - y_1 = m(x - x_1)$

$$y - (-3) = \tfrac{3}{4}(x-2)$$
$$4(y+3) = 3(x-2)$$
$$4y + 12 = 3x - 6$$
$$4y = 3x - 18$$

You can see more examples like this using the Flash resource **Parallel lines**.

An alternative approach to the above examples is to put the formula for m into the straight line equation to obtain

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

and then make the substitutions. This is equivalent to the first method, but does not involve calculating m separately first.

For further practice in examples like the one above, try the interactive resource **The equation of a line between two points**.

The intersection of two lines

The point of intersection of two lines is found by solving the equations of the lines simultaneously. This can be done in a variety of ways. When both equations are given in the form $y = \ldots$ then equating the right hand sides is a good approach (see below). If both equations are not in this form, you can re-arrange them into this form first, then apply the same method. Alternatively, you can use the elimination method if the equations are in an appropriate form.

Example 7

Find the point of intersection of the lines $y = 3x - 2$ and $y = 5x - 8$.

Solution

$$3x - 2 = 5x - 8$$
$$\Rightarrow -2 = 2x - 8$$
$$\Rightarrow 6 = 2x$$
$$\Rightarrow x = 3$$

Substitute into one of the equations to find y

Substituting $x = 3$ into $y = 3x - 2$ gives $y = 3 \times 3 - 2 = 7$

The point of intersection is (3, 7).

Check that (3, 7) satisfies the second equation.

Example 8
Find the point of intersection of the lines $3y + 2x = 4$ and $7y + 3x = 2$.

Solution 1
Using the elimination method:

$$\begin{aligned} 3y + 2x &= 4 \quad \times 3 \qquad 9y + 6x = 12 \\ 7y + 3x &= 2 \quad \times 2 \qquad 14y + 6x = 4 \end{aligned}$$

Subtracting: $-5y = 8$

$y = -\frac{8}{5}$

Substituting into first equation: $3 \times -\frac{8}{5} + 2x = 4$

$2x = 4 + \frac{24}{5} = \frac{44}{5}$

$x = \frac{22}{5}$

Intersection point is $\left(\frac{22}{5}, -\frac{8}{5}\right)$.

Solution 2

$3y + 2x = 4 \quad \Rightarrow 3y = 4 - 2x \quad \Rightarrow y = \dfrac{4 - 2x}{3}$

$7y + 3x = 2 \quad \Rightarrow 7y = 2 - 3x \quad \Rightarrow y = \dfrac{2 - 3x}{7}$

Equating the two expressions for y: $\dfrac{4 - 2x}{3} = \dfrac{2 - 3x}{7}$

$7(4 - 2x) = 3(2 - 3x)$

$28 - 14x = 6 - 9x$

$22 = 5x$

$x = \frac{22}{5}$

Substituting into $y = \dfrac{4 - 2x}{3} = \frac{4}{3} - \frac{2}{3} \times \frac{22}{5} = \frac{4}{3} - \frac{44}{15} = \frac{20 - 44}{15} = -\frac{24}{15} = -\frac{8}{5}$

Intersection point is $\left(\frac{22}{5}, -\frac{8}{5}\right)$.

You can see more examples like this using the Flash resource **Intersection of two lines**. Use algebra to work out the intersection point, and then check your answer.

Perpendicular lines

If two lines with gradients m_1 and m_2 are perpendicular, then $m_1 m_2 = -1$

Example 9
P is the point (-3, 7). Q is the point (5, 1).
Calculate
(i) the gradient of PQ

(ii) the equation of a line perpendicular to PQ which passes through the point (2, -1).

Solution

(i) Gradient of PQ = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-7}{5-(-3)} = \dfrac{-6}{8} = -\dfrac{3}{4}$

(ii) When two lines are perpendicular $m_1 m_2 = -1$.

So $-\dfrac{3}{4}m_2 = -1 \quad \Rightarrow m_2 = \dfrac{4}{3}$

The gradient of a line perpendicular to PQ is $\dfrac{4}{3}$.

The line passes through the point (2, -1)

The equation of the line is $y-(-1) = \frac{4}{3}(x-2)$

$$3(y+2) = 4(x-2)$$

$$3y + 6 = 4x - 8$$

$$3y = 4x - 14$$

For further examples look at the Flash resource **Perpendicular lines**

For further practice in examples like the one above, try the interactive questions **The gradient of a perpendicular**.