Further Pure 1

Complex Numbers
Exercise G

9(i) $\alpha = 1+4j$

$\alpha^2 = (1+4j)^2 = 1+8j-16 = -15+8j$

$\alpha^3 = (1+4j)(-15+8j) = -15-52j-32 = -47-52j$

(ii) $z^3+5z^2+kz+m=0$

$-47-52j+5(-15+8j)+k(1+4j)+m=0$ substituting α for z

$-47-52j-75+40j+k+4kj+m=0$

$(-47-75+k+m)+(-52+40+4k)j=0$

$(-122+k+m)+(-12+4k)j=0$

Equating imaginary parts: $-12+4k=0 \Rightarrow k=3$

Equating real parts: $-122+k+m=0 \Rightarrow m = 122-k$
$= 119$

(iii) $1+4j$ is a root $\Rightarrow 1-4j$ is also a root

$(z-1-4j)$ and $(z-1+4j)$ are factors

$$
\begin{aligned}
(z-1-4j)(z-1+4j) &= (z-1)^2+16 \\
&= z^2-2z+1+16 \\
&= z^2-2z+17 \quad \text{is a factor}
\end{aligned}
$$

$z^3+5z^2+3z+119=0$

$(z^2-2z+17)(z+7)=0$

The roots are $1+4j$, $1-4j$ and -7.

$\arg(-7) = \pi$

$\arg(1+4j) = \arctan\left(\frac{4}{1}\right) = 1.33$ $1+4j$ is in the first quadrant

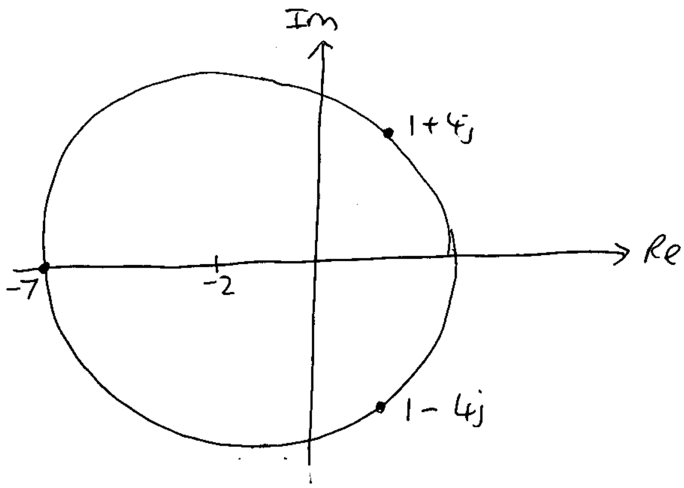
$\arg(1-4j) = \arctan\left(\frac{-4}{1}\right) = -1.33$ $1-4j$ is in the fourth quadrant

(iv) $|-7+2| = |-5| = 5$

$|1+4j+2| = |3+4j| = \sqrt{(3^2+4^2)} = 5$

$|1-4j+2| = |3-4j| = \sqrt{(3^2+4^2)} = 5$

All three roots satisfy $|z+2| = 5$



$|z+2| = 5$ is a circle centre -2, radius 5