Further Pure 1

Complex Numbers
Exercise G

3. $z = 1 - j$

$$z^3 + pz^2 + qz + 12 = 0$$
$$(1-j)^3 + p(1-j)^2 + q(1-j) + 12 = 0$$
$$1 - 3j + 3j^2 - j^3 + p(1 - 2j + j^2) + q(1-j) + 12 = 0$$
$$1 - 3j - 3 + j + p(1 - 2j - 1) + q(1-j) + 12 = 0$$
$$-2 - 2j - 2pj + q - qj + 12 = 0$$
$$(-2 + q + 12) + (-2 - 2p - q)j = 0$$
$$10 + q + (-2 - 2p - q)j = 0$$

Equating real parts $\Rightarrow$ $10 + q = 0 \Rightarrow q = -10$

Equating imaginary parts $\Rightarrow$ $-2 - 2p - q = 0$
$\Rightarrow$ $-2 - 2p + 10 = 0$
$\Rightarrow$ $2p = 8 \Rightarrow p = 4$

$1 + j$ is also a root, so $(z - 1 + j)$ and $(z - 1 - j)$ are factors.

$$(z - 1 + j)(z - 1 - j) = (z-1)^2 + 1$$
$$= z^2 - 2z + 1 + 1$$
$$= z^2 - 2z + 2$$

$$z^3 + 4z^2 - 10z + 12 = 0$$
$$(z^2 - 2z + 2)(z + 6) = 0$$ by inspection

$$z = -6,\ 1 - j,\ 1 + j$$