

Further Pure 1

Complex Numbers

Exercise G

1. $z = 2 + j$

$$\begin{aligned}z^3 - z^2 - 7z + 15 &= (2+j)^3 - (2+j)^2 - 7(2+j) + 15 \\&= 8 + 3 \times 4j + 3 \times 2j^2 + j^3 - (4 + 4j + j^2) \\&\quad - 14 - 7j + 15 \\&= 8 + 12j - 6 - j - 4 - 4j + 1 - 14 - 7j + 15 \\&= (8 - 6 - 4 + 1 - 14 + 15) + (12 - 1 - 4 - 7)j \\&= 0\end{aligned}$$

so $2 + j$ is a root.

$2 - j$ is also a root, so $(z - 2 - j)$ and $(z - 2 + j)$ are both factors.

$$\begin{aligned}(z - 2 - j)(z - 2 + j) &= (z - 2)^2 - j^2 \\&= z^2 - 4z + 4 + 1 \\&= z^2 - 4z + 5 \quad \text{is a factor}\end{aligned}$$

$$z^3 - z^2 - 7z + 15 = 0$$

$$(z^2 - 4z + 5)(z + 3) = 0 \quad \text{by inspection}$$

The other factor is $z + 3 \Rightarrow$ the other root is $z = -3$

The roots are $2 + j$, $2 - j$ and -3 .