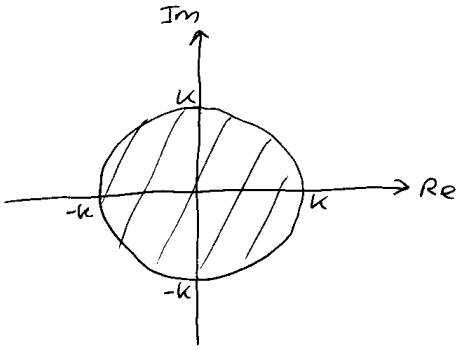
Further Pure 1

Complex Numbers
Exercise F

4 (i)



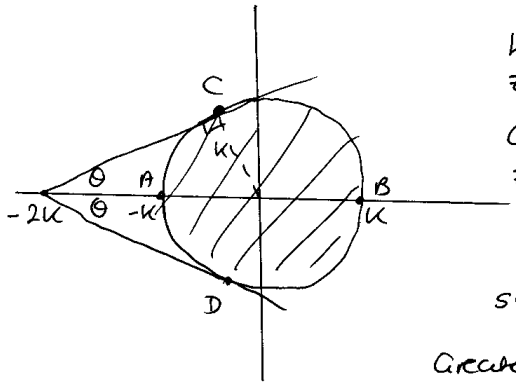
$|z+k|$ is the distance between the point z and the point $-k$.

From the diagram, the minimum value of $|z+k| = 0$ (when $z = -k$) and the maximum value of $|z+k| = 2k$ (when $z = k$). So $0 \leq |z+k| \leq 2k$

$\arg(z+k)$ is the angle between a line connecting z to the point $-k$ and the real axis. When $z = -k$ $\arg(z+k)$ is undefined. When z is on the perimeter of the circle infinitely close to $-k$ and above it, then $\arg(z+k)$ is just less than $\frac{\pi}{2}$. When z is on the perimeter of the circle infinitely close to $-k$ and below it, then $\arg(z+k)$ is just greater than $-\frac{\pi}{2}$.

$$-\frac{\pi}{2} < \arg(z+k) < \frac{\pi}{2}$$

(ii)



Least value of $|z+2k|$ is when z is at A, when $|z+2k| = k$

Greatest value of $|z+2k|$ is when z is at B, when $|z+2k| = 3k$

$$k \leq |z+2k| \leq 3k$$

$$\sin\theta = \frac{k}{2k} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

Greatest value of $\arg(z+2k)$ is when z is at C, when $\arg(z+2k) = \frac{\pi}{6}$

Least value of $\arg(z+2k)$ is when z is at D, when $\arg(z+2k) = -\frac{\pi}{6}$

$$-\frac{\pi}{6} \leq \arg(z+2k) \leq \frac{\pi}{6}$$