

Further Pure Mathematics 1

Complex Numbers Unit 3

Hints

Exercise 2F

Question 1 Hint level 1

Look at Example 2.8. All these questions are similar.

Question 2 Hint level 1

Draw the set of points $|z - 8j| \leq 4$.

Imagine a line from the origin, which starts on the real axis and rotates anticlockwise about the origin. Draw the line when it first touches the set of points $|z - 8j| \leq 4$. What is the value of $\arg z$ at this point? This is the minimum value of $\arg z$. Then imagine the line continuing to rotate until it no longer touches the set of points $|z - 8j| \leq 4$. What is the value of $\arg z$ just before this happens? This is the maximum value of $\arg z$.

Question 3 Hint level 1

- (ii) The first inequality represents a circle and its interior. The second represents the area between the lines representing $\arg z = \frac{1}{2}\pi$ and $\arg z = \frac{2}{3}\pi$.
- (iii) $|z - w|$ is the distance between z and w . Look at your diagram and see which point in the shaded area is furthest from w .

Question 4 Hint level 1

- (i) Draw a diagram to show the set of points for which $|z| \leq k$.
 $|z + k|$ is the distance between the point z and the point $-k$. Look at your diagram to find the greatest and least possible distances of z in the shaded area from $-k$.
 $\arg(z + k)$ is the angle between a line from $-k$ to z and the real axis. Look at your diagram to find the greatest and least possible angles.
- (ii) $|z + 2k|$ is the distance between the point z and the point $-2k$. Look at your diagram to find the greatest and least possible distances of z in the shaded area from $-2k$.
 $\arg(z + 2k)$ is the angle between a line from $-2k$ to z and the real axis. Look at your diagram to find the greatest and least possible angles.