Further Pure Mathematics 1

Complex Numbers

Section 2: The Argand diagram

Exercise 2D Hints level 2

Question 1 Hint level 2

Any set of points of the form $|z - (a + bj)| = r$ represents a circle, centre $a + bj$, radius r.

Remember that $|z - (a + bj)| \leq r$ represents the circle and its interior, and $|z - (a + bj)| < r$ represents just the interior (so the circle itself is drawn as a dotted line). Similarly $|z - (a + bj)| \geq r$ represents the circle and its exterior, and $|z - (a + bj)| > r$ represents just the exterior.

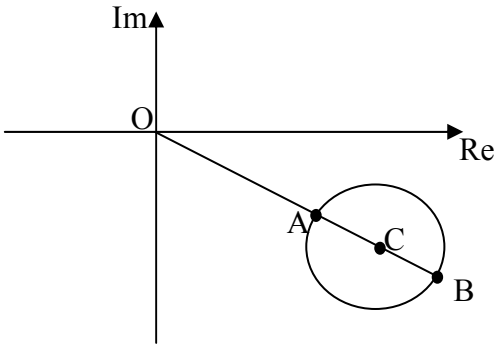
(vi) What is a circle with radius 0?

(vii) In this case the radius is between 2 and 3. Think about how to draw this.

(viii) Think of several points for which Re(z) = -2.

Question 2 Hint level 2

The diagram shows the circle $|z - 12 + 5j| \leq 7$.

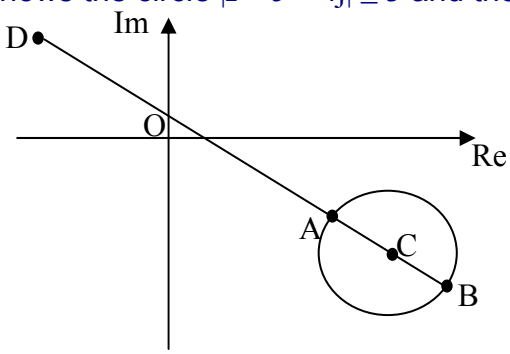


Since $|z|$ represents the distance of the point z from the origin, the smallest possible value of $|z|$ must be the distance OA, which is OC – 7, and the largest possible value of $|z|$ must be the distance OB, which is OC + 7.

Question 3 Hint level 2

This is similar to question 2, but this time you are looking for the greatest and least values of $|z + 3 - 2j|$, which means the greatest and least possible distances of the point z from the point $-3 + 2j$.

The diagram shows the circle $|z - 5 + 4j| \leq 3$ and the point D representing $-3 + 2j$.



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The least possible value of $|z + 3 - 2j|$ is the distance DA, which is the distance DC – 3, and the greatest possible value of $|z + 3 - 2j|$ is the distance DB, which is the distance DC + 3.

Question 4 Hint level 2

You need to decide whether or not the circle $|z + 4 + 2j| = 2$ lies completely within the circle $|z - 2 + j| = 10$. If it does, a point cannot be inside the smaller circle and outside the larger circle at the same time, so there are no points which satisfy both conditions. To find out, work out the distance between the centres of the two circles.

Question 5 Hint level 2

Any set of points of the form $|z - (a + bj)| = |z - (c + dj)|$ represents the points which are equidistant from the point $a + bj$ and the point $c + dj$; i.e. the points on the perpendicular bisector of the points $a + bj$ and $c + dj$. If the set of points involves ≤ or ≥ instead of =, the set of points includes the perpendicular bisector and all the points on one side of it.

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