Further Pure 1

Complex Numbers
Exercise A

2(i) $z^2 + 2z + 2 = 0$

$$z = \frac{-2 \pm \sqrt{(2^2 - 4\times 2)}}{2}$$

$$= \frac{-2 \pm \sqrt{(4-8)}}{2}$$

$$= \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm 2j}{2}$$

$$= -1 \pm j$$

Check: $z = -1+j$ $\quad z^2 + 2z + 2 = (-1+j)^2 + 2(-1+j) + 2$

$= 1 - 2j + j^2 - 2 + 2j + 2$

$= 1 - 2j - 1 - 2 + 2j + 2$

$= 0$

$z = -1-j$ $\quad z^2 + 2z + 2 = (-1-j)^2 + 2(-1-j) + 2$

$= 1 + 2j + j^2 - 2 - 2j + 2$

$= 1 + 2j - 1 - 2 - 2j + 2$

$= 0$

(ii) $z^2 - 2z + 5 = 0$

$$z = \frac{2 \pm \sqrt{(4 - 4\times 5)}}{2}$$

$$= \frac{2 \pm \sqrt{-16}}{2}$$

$$= \frac{2 \pm 4j}{2}$$

$$= 1 \pm 2j$$

Check: $z = 1+2j$ $\quad z^2 - 2z + 5 = (1+2j)^2 - 2(1+2j) + 5$

$= 1 + 4j + 4j^2 - 2 - 4j + 5$

$= 1 + 4j - 4 - 2 - 4j + 5$

$= 0$

$z = 1-2j$ $\quad z^2 - 2z + 5 = (1-2j)^2 - 2(1-2j) + 5$

$= 1 - 4j + 4j^2 - 2 + 4j + 5$

$= 1 - 4j - 4 - 2 + 4j + 5$

$= 0$

(iii) $z^2 - 4z + 13 = 0$

$$z = \frac{4 \pm \sqrt{(16 - 4 \times 13)}}{2}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$= \frac{4 \pm 6j}{2}$$

$$= 2 \pm 3j$$

Check: $z = 2+3j$ $\quad z^2 - 4z + 13 = (2+3j)^2 - 4(2+3j) + 13$

$= 4 + 12j + 9j^2 - 8 - 12j + 13$

$= 4 + 12j - 9 - 8 - 12j + 13$

$= 0$

$z = 2-3j$ $\quad z^2 - 4z + 13 = (2-3j)^2 - 4(2-3j) + 13$

$= 4 - 12j + 9j^2 - 8 + 12j + 13$

$= 4 - 12j - 9 - 8 + 12j + 13$

$= 0$

(iv) $z^2 + 6z + 34 = 0$

$$z = \frac{-6 \pm \sqrt{(36 - 4 \times 34)}}{2}$$

$$= \frac{-6 \pm \sqrt{-100}}{2}$$

$$= \frac{-6 \pm 10j}{2}$$

$$= -3 \pm 5j$$

Check: $z = -3 + 5j$ $\quad z^2 + 6z + 34 = (-3+5j)^2 + 6(-3+5j) + 34$

$= 9 - 30j + 25j^2 - 18 + 30j + 34$

$= 9 - 30j - 25 - 18 + 30j + 34$

$= 0$

$z = -3 - 5j$ $\quad z^2 + 6z + 34 = (-3-5j)^2 + 6(-3-5j) + 34$

$= 9 + 30j + 25j^2 - 18 - 30j + 34$

$= 9 + 30j - 25 - 18 - 30j + 34$

$= 0$

(v) $4z^2 - 4z + 17 = 0$

$$z = \frac{4 \pm \sqrt{(16 - 4 \times 4 \times 17)}}{2 \times 4}$$

$$= \frac{4 \pm \sqrt{-256}}{8}$$

$$= \frac{4 \pm 16j}{8}$$

$$= \frac{1}{2} \pm 2j$$

Check: $z = \frac{1}{2} + 2j$ $\quad 4z^2 - 4z + 17 = 4(\frac{1}{2} + 2j)^2 - 4(\frac{1}{2} + 2j) + 17$

$= 4(\frac{1}{4} + 2j + 4j^2) - 2 - 8j + 17$

$= 1 + 8j - 16 - 2 - 8j + 17$

$= 0$

$z = \frac{1}{2} - 2j$ $\quad 4z^2 - 4z + 17 = 4(\frac{1}{2} - 2j)^2 - 4(\frac{1}{2} - 2j) + 17$

$= 4(\frac{1}{4} - 2j + 4j^2) - 2 + 8j + 17$

$= 1 - 8j - 16 - 2 + 8j + 17$

$= 0$

(vi) $z^2 + 4z + 6 = 0$

$z = \dfrac{-4 \pm \sqrt{(16 - 4 \times 6)}}{2}$

$= \dfrac{-4 \pm \sqrt{-8}}{2}$

$= \dfrac{-4 \pm 2\sqrt{2}j}{2}$

$= -2 \pm \sqrt{2}j$

Check: $z = -2 + \sqrt{2}j$ $\quad z^2 + 4z + 6 = (-2 + \sqrt{2}j)^2 + 4(-2 + \sqrt{2}j) + 6$

$= 4 - 4\sqrt{2}j + 2j^2 - 8 + 4\sqrt{2}j + 6$

$= 4 - 4\sqrt{2}j - 2 - 8 + 4\sqrt{2}j + 6$

$= 0$

$z = -2 - \sqrt{2}j$ $\quad z^2 + 4z + 6 = (-2 - \sqrt{2}j)^2 + 4(-2 - \sqrt{2}j) + 6$

$= 4 + 4\sqrt{2}j + 2j^2 - 8 - 4\sqrt{2}j + 6$

$= 4 + 4\sqrt{2}j - 2 - 8 - 4\sqrt{2}j + 6$

$= 0$