Core 1

Chapter assessment

Coordinate Geometry

1. Find the coordinates of the points where the line $5y+2x+10=0$ meets the axes and hence sketch the line.

2. Describe fully the curve whose equation is $x^2+y^2=4$.

3. The coordinates of two points are A(-1,-3) and B(5,7). Calculate the equation of the perpendicular bisector of AB.

4. Show that the line $y=3x-10$ is a tangent to the circle $x^2+y^2=10$.

5. Find the coordinates of the points of intersection of the curves $y=x^2-5$ and $y=3-x^2$

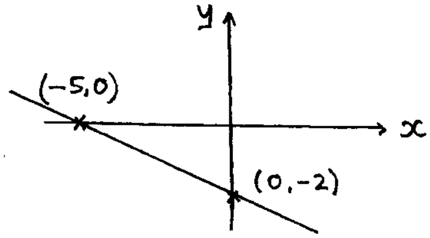
6. The coordinates of four points are P(-2,-1), Q(6,3), R(9,2) and S(1,-2).
   (i) Calculate the gradients of the lines PQ, QR, RS and SP.
   (ii) What name is given to the quadrilateral PQRS?
   (iii) Calculate the length SR.
   (iv) Show that the equation of SR is $2y=x-5$ and find the equation of the line L through Q perpendicular to SR.
   (v) Calculate the coordinates of the point T where the line L meets SR.
   (vi) Calculate the area of the quadrilateral PQRS.

7. AB is the diameter of a circle. A is (1,3) and B(7,-1).
   (i) Find the coordinates of the centre of the circle.
   (ii) Find the radius of the circle.
   (iii) Find the equation of the circle.
   (iv) The line $5y=x+14$ cuts the circle at A and again at a second point D. Calculate the coordinates of D.
   (v) Prove that the line AB is perpendicular to the line CD.

Core 1

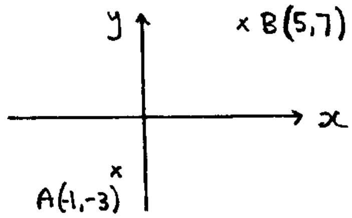
Coordinate geometry chapter assessment solutions

1. When $x = 0$, $5y + 2(0) + 10 = 0$ so $y = -2$ giving $(0, -2)$.
When $y = 0$, $5(0) + 2x + 10 = 0$ so $x = -5$ giving $(-5, 0)$.



2. The curve $x^2 + y^2 = 4$ is a circle, centre $(0,0)$ and radius $\sqrt{4} = 2$.

3.



Midpoint M of AB is $\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right) = \left(\frac{-1+5}{2}, \frac{-3+7}{2}\right) = (2, 2)$

Gradient of AB $= \frac{y_B - y_A}{x_B - x_A} = \frac{7-(-3)}{5-(-1)} = \frac{10}{6} = \frac{5}{3}$

Using $m_1 m_2 = -1$ gradient of $\perp^r$ bisector is $-\frac{3}{5}$.

Equation of $\perp^r$ bisector is $y = -\frac{3}{5}x + c$ and subst. $M(2,2)$

$2 = -\frac{3}{5}(2) + c$ giving $c = \frac{16}{5}$

$\therefore \quad y = -\frac{3}{5}x + \frac{16}{5}$

(or $5y + 3x = 16$)

4 Subst. $y = 3x - 10$ into $x^2 + y^2 = 10$

$$x^2 + (3x-10)^2 = 10$$
$$x^2 + 9x^2 - 60x + 100 = 10$$
$$10x^2 - 60x + 90 = 0$$
$$x^2 - 6x + 9 = 0$$
$$(x-3)(x-3) = 0$$

When $x = 3$, $y = 3(3) - 10 = -1$ giving $(3, -1)$.

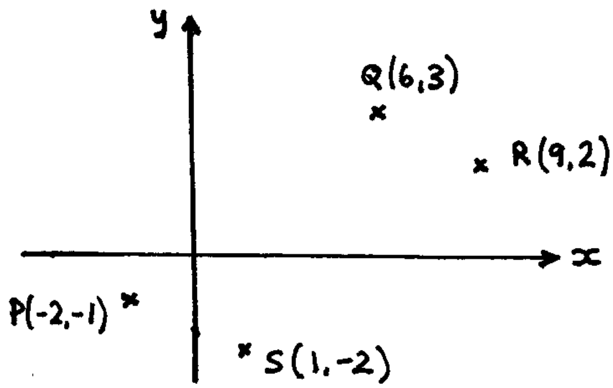
Since $x = 3$ is a repeated root the line $y = 3x - 10$ is a tangent to the circle $x^2 + y^2 = 10$ at $(3, -1)$.

5 Solve $y = x^2 - 5$ and $y = 3 - x^2$ simultaneously.

$$x^2 - 5 = 3 - x^2$$
$$2x^2 = 8$$
$$x^2 = 4$$
$$x = \pm 2$$

When $x = +2$, $y = (2)^2 - 5 = -1$ giving $(2, -1)$
When $x = -2$, $y = (-2)^2 - 5 = -1$ giving $(-2, -1)$
$\therefore$ the points of intersection are $(2, -1)$ and $(-2, -1)$.

⑥



(i) Using the gradient formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m_{PQ} = \frac{3-(-1)}{6-(-2)} = \frac{4}{8} = \frac{1}{2}$$

$$m_{QR} = \frac{2-3}{9-6} = -\frac{1}{3}$$

$$m_{RS} = \frac{-2-2}{1-9} = \frac{-4}{-8} = \frac{1}{2}$$

$$m_{SP} = \frac{-1-(-2)}{-2-1} = \frac{1}{-3} = -\frac{1}{3}$$

(ii) Since $m_{PQ} = m_{RS}$, PQ and RS are parallel.
Also $m_{QR} = m_{SP}$ so QR and SP are parallel.
$\therefore$ PQRS is a parallelogram.

(iii) $$SR = \sqrt{(x_S - x_R)^2 + (y_S - y_R)^2}$$
$$= \sqrt{(1-9)^2 + (-2-2)^2}$$
$$= \sqrt{64 + 16}$$
$$= \sqrt{80} \quad (\text{or } 4\sqrt{5})$$

(iv) Consider the straight line equation $2y = x - 5$.

When $x = 1$, $2y = (1) - 5$ giving $y = -2$ $\therefore S(1,-2)$ lies on line.

When $x = 9$, $2y = (9) - 5$ giving $y = 2$ $\therefore R(9,2)$ lies on line.

$\therefore$ the equation of SR is $2y = x - 5$
(since two points fix the line)

Gradient of SR $= \frac{1}{2}$ (from $y = \frac{1}{2}x - \frac{5}{2}$)

Using $m_1 m_2 = -1$, gradient of perpendicular is -2.

Let perpendicular line be $y = -2x + c$ and subst. $Q(6,3)$

$3 = -2(6) + c$ giving $c = 15$

The equation of the perpendicular line is $y = -2x + 15$

(v) Solving $2y = x - 5$ and $y = -2x + 15$ simultaneously

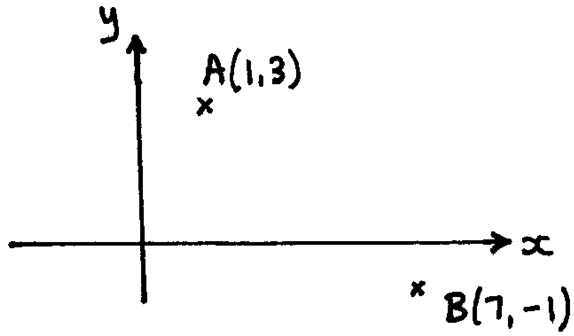
$$x - 5 = 2(-2x + 15)$$
$$x - 5 = -4x + 30$$
$$5x = 35$$
$$x = 7$$

and substituting, $y = 1$ $\therefore$ T is $(7, 1)$

(vi) Area of PQRS $= SR \times QT$

$$= 4\sqrt{5} \times \sqrt{(7-6)^2 + (1-3)^2}$$
$$= 4\sqrt{5} \times \sqrt{1+4}$$
$$= 4\sqrt{5} \times \sqrt{5}$$
$$= 20 \text{ square units}$$

7



(i) Centre, C, is $\left(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2}\right) = \left(\frac{1+7}{2}, \frac{3+(-1)}{2}\right) = (4, 1)$

(ii) Radius, AC $= \sqrt{(x_A-x_C)^2+(y_A-y_C)^2}$

$= \sqrt{(1-4)^2+(3-1)^2}$

$= \sqrt{9+4} = \sqrt{13}$

(iii) Using $(x-a)^2+(y-b)^2=r^2$

with $a=4$, $b=1$ and $r=\sqrt{13}$ gives

$(x-4)^2+(y-1)^2=13$

(iv) $5y = x + 14$ can be written $x = 5y - 14$.

Substitute this into the equation of the circle.

$$((5y-14)-4)^2 + (y-1)^2 = 13$$
$$(5y-18)^2 + (y-1)^2 = 13$$
$$25y^2 - 180y + 324 + y^2 - 2y + 1 = 13$$
$$26y^2 - 182y + 312 = 0$$
$$(\div 26) \qquad y^2 - 7y + 12 = 0$$
$$(y-3)(y-4) = 0$$
$$y = 3 \text{ or } y = 4$$

When $y=3$, $x = 5(3) - 14 = 1$ giving $(1,3)$ which is A.

When $y=4$, $x = 5(4) - 14 = 6$ giving $(6,4)$ which is D.

(v) Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$ so

$$m_{AB} = \frac{-1-3}{7-1} = \frac{-4}{6} = -\frac{2}{3}$$

$$m_{CD} = \frac{4-1}{6-4} = \frac{3}{2}$$

Since $m_{AB} \times m_{CD} = -\frac{2}{3} \times \frac{3}{2} = -1$

then AB is perpendicular to CD.