

## Section 1: Algebra

### Glossary

#### Algebraic Fraction

A fraction that contains at least one variable term in either the numerator or the denominator or in both.

E.g.  $\frac{6x}{5}$ ;  $\frac{3x-2}{x^2+3}$

Algebraic fractions can be manipulated in the same way as numerical fractions and obey the same rules.

#### Binomial Expansion

The expansion of an expression of the form  $(a+b)^n$ . It is called binomial because it involves two variables. (“bi” means two)

#### Coefficient

The number in front of a variable term. E.g. the coefficient of  $x^2$  in the expression  $3x^2 + 6x - 2$  is 3.

#### Common Factor

A number, letter or bracket that appears in every term of an expression.

E.g.  $5x - 25$  has a common factor of 5;  $x^3 - 2x$  has a common factor of  $x$ ;  $x(x-3) + 5(x-3)$  has a common factor of  $(x-3)$ .

#### Completing the Square

A process whereby a quadratic expression is written in the form of  $(x+a)^2 + b$ .

#### Constant

This is the term in an expression that does not involve a variable. E.g.  $ax + b$  has a constant term  $b$ ;  $3x^2 - 7$  has a constant term of  $-7$ .

#### Cubic Expression

This is an expression where the highest power of the variable is 3. E.g.

$$x^3 - 5x^2 + 2x - 1.$$

#### Difference of Two Squares

This is a particular type of expression involving terms in  $x^2$  and a constant term. E.g.  $x^2 - 25$  is the difference of two squares as both components are perfect squares. The factors of  $x^2 - 25$  are  $(x+5)(x-5)$

#### Discriminant

In a quadratic equation of the form  $ax^2 + bx + c = 0$ ,  $b^2 - 4ac$  is the discriminant.

It gives information about the roots of the equation. If  $b^2 - 4ac$  is positive then the equation has two real roots; if it is zero then the equation has one repeated root; and if it is negative, the equation has no real roots.

## Dividend

In any division the number or expression being divided is the dividend. E.g.  $60$

$\div 5$  has a dividend of  $60$ ;  $\frac{x^2 + 7x - 9}{(x + 3)}$  has a dividend of  $x^2 + 7x - 9$ .

## Divisor

In any division the number or expression you are dividing by is called the divisor.

E.g.  $60 \div 5$  has a divisor of  $5$ , and  $\frac{x^2 + 7x - 9}{(x + 3)}$  has a divisor of  $(x + 3)$

## Factor

Any number or expression that divides into another without remainder is a factor of the dividend. E.g.  $55 \div 11 = 5$ , hence  $11$  is a factor of  $55$ ;

$\frac{(x^2 + 5x + 6)}{(x + 3)}$  gives a remainder of zero therefore,  $(x + 3)$  is a factor of  $(x^2 + 5x + 6)$ .

## Factor Theorem

If  $(x - a)$  is a factor of a polynomial  $f(x)$  then  $f(a) = 0$ . Conversely if  $f(a) = 0$  then  $(x - a)$  is a factor of  $f(x)$ . The theorem is useful in factorising polynomials and in solving polynomial equations. It is a special case of the remainder theorem.

## Factorial

The factorial of a number is written as  $n!$  such that  $n! = n(n - 1)(n - 2)\dots\dots 1$

## Index (indices)

The index of a number or variable is the power that that number or variable is to be raised to. E.g.  $x^4$  has an index of  $4$ . Indices is the plural of index.

## Integer

An integer is a whole number (or counting number). It may be negative.

## Linear Expression

Is an expression in which the highest power of the variable is one.  $ax + b$  is a linear expression.  $y = ax + b$  represents a straight line, hence the description linear.

## Linear Equation

An equation in which the highest power of the variable is one.

E.g.  $3x + 2 = 0$ ;  $5x - 4 = 3x + 1$ .

## Order of a polynomial

The highest power present of the variable gives the power of the polynomial.

E.g.  $x^3 - 4x + 2$  has an order of  $3$ .

## Parabola

The curve that is produced when the graph of a quadratic expression is drawn. It is also the path traced by any projectile.

## Pascal's Triangle

The coefficients of the expansion of  $(a + b)^n$  when written in the form of a triangle with each row representing a different value of  $n$  is called Pascal's triangle named after the French mathematician Blaise Pascal. Each number is the sum of the two numbers immediately above it.

## Perfect Square

A quadratic is a perfect square if it can be written as the product of two identical linear expressions. E.g.  $x^2 + 4x + 4 = (x + 2)^2$  and hence  $x^2 + 4x + 4$  is a perfect square.

## Polynomial

An expression involving a number of terms with positive integer powers of one variable only, it may also include a constant term.

E.g. a polynomial may involve terms such as  $x^5$  or  $x^7$  but cannot involve terms such as  $\sqrt{x}$  or  $xy$  or  $\frac{1}{x}$ .

## Product

The answer when two or more numbers or expressions are multiplied together.

$3 \times 5$  has a product of 15;  $(x + 3)(x - 2)$  has a product of  $x^2 + x - 6$ .

## Quadratic Expression

An expression in which the highest power of the variable is 2

## Quadratic formula

This is the formula that can be used to solve a quadratic equation

$$ax^2 + bx + c = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## Quartic Expression

A polynomial in which the highest power of the variable is 4

## Quintic Expression

A polynomial in which the highest power of the variable is 5.

## Quotient

In a division calculation, whether algebraic or numeric, the result of the

division is called the quotient.  $\frac{23}{5} = 4$  remainder 3. The quotient is 4.

## Rational Numbers

These are numbers that can be represented by a fraction in its simplest form. i.e. the fraction cannot be cancelled down even further. (Irrational numbers cannot be written in this way and include numbers such as  $\pi$ ,  $\sqrt{2}$  etc.)

## Real Numbers

All of the numbers you have met up to now, or will meet on a standard Maths Additional, AS or A level course will be real numbers. They are numbers that can be represented on a number line and include both the rational and irrational numbers. Integers are rational numbers.

## Remainder

In any division calculation, whether numeric or algebraic, the number or expression left over is the remainder. In an algebraic division the order of the remainder will be at least one less than the order of the divisor.

## Remainder Theorem

This states that if a polynomial is divided by  $(x - a)$  then the remainder will be  $f(a)$ .

## Roots

The roots of an equation are the values that the variable can take; i.e.

$$x^2 + 5x + 6 = 0$$

$$\Rightarrow (x + 3)(x + 2) = 0$$

The roots of the equation are  $-2$  and  $-3$ .

$$\Rightarrow x = -3 \text{ or } x = -2$$

## Simultaneously Equations

A set of two or more equations involving two or more variables. We need the same number of equations as there are of variables before we can obtain a solution.

## Variable

An unknown quantity represented by a letter such as  $x$ .

## Vertex

The turning point on a quadratic is sometimes called the vertex.