Additional Mathematics

Algebra II – techniques 2

Notes and examples

Algebraic Fractions

There are other notes on this in the earlier chapter but it is worth bearing in mind that if something doesn't work with numbers then it will not work when using letters. Cancelling is a major source of error.

Try cancelling the 'n' in $\frac{\sin x}{n}$. Do you really think that 6 is the answer?

The above was a very silly example but it is often tempting to cancel when you shouldn't. E.g. $\frac{2x+3}{3x-1}$ If we try to cancel x we end up with $\frac{5}{2}$. But this is only true if $x = 1$, it is untrue for all other values.

There are times when you may need to factorise before you can see if anything will cancel.

Example 1

Simplify as far as possible:

i) $\frac{x^2-5x+6}{x^2+x-12}$

ii) $\frac{2n+3}{4n^2-9} \times \frac{2n^2-n-3}{n-5}$

Solutions

i) $$\frac{\cancel{(x-3)}(x-2)}{(x+4)\cancel{(x-3)}} = \frac{(x-2)}{(x+4)}$$

ii) $$\frac{\cancel{(2n+3)}}{\cancel{(2n+3)}\cancel{(2n-3)}} \times \frac{\cancel{(2n-3)}(n+1)}{(n-5)} = \frac{(n+1)}{(n-5)}$$

When adding or subtracting, a common denominator is required. The lowest common denominator is best as it saves work later.

Example 2

Simplify: $\frac{3x}{x^2-9} + \frac{x}{x^2-x-6}$

Solution

$$\frac{3x}{(x-3)(x+3)}+\frac{x}{(x-3)(x+2)}$$

We don't need $(x-3)^2$

Lowest common denominator is: $(x-3)(x+3)(x+2)$

$$=\frac{3x(x+2)+x(x+3)}{(x+3)(x-3)(x+2)}$$

$$=\frac{4x^2+9x}{(x+3)(x-3)(x+2)}$$

These techniques are used to solve equations.

Example 3

Solve: $\frac{x+1}{2x-3}-\frac{1}{x-3}=0$

Solution

The lowest common denominator is $(2x-3)(x-3)$ $\therefore$ Multiply through by this.

$(x+1)(x-3)-(2x-3)=0$

Note: brackets kept on to avoid errors in sign here

$\Rightarrow \quad x^2-2x-3-2x+3=0$

$\Rightarrow \quad x^2-4x=0$

$\Rightarrow \quad x(x-4)=0 \qquad$ i.e. $x=0$ or $x=4$